POLITICAL ECONOMY OF HETEROGENEOUS POPULATIONS: OPTIMAL EQUILIBRIUM PLUNDER

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Abstract. We study the political economy of societies with heterogeneous populations. Agents in the economy decide on policy issues with majority voting (as in (Meltzer and Richard (1981))). But in our model the role of the political class is made explicit, as in the tradition of the Public Choice school (Romer (1988)). Thus , and the equilibria are determined by the joint action of the choices of agents and voters and that of the political class.

1. Introduction

The topics studied in these notes are complex, so we proceed in steps.

In section (2) we define the general setup of a problem in which voters decide the distribution of income through taxation ny majority voting and the political class decides the composition of the population (for example through immigration). In section (3) we analyze the special case in which citizens do not choose effort in production.

In later sections we will examine:

(1) The case in which the agents provide effort, so the effect of taxes on effort becomes relevant;
(2) The non static problem in which people live over different periods and/or care about the children, and so the motivation of upward mobility becomes relevant;
(3) The billion/trillion/many trillions note on the floor that we just need to pick up through immigration;
(4) the issue of the optimal organization in countries of a world population when individuals and groups differ systematically in their characteristics (and so not just intelligence, but, say, discount factor, risk aversion, conscientiousness and so on).

2. Static Case: setup

We first consider the static (time-independent) case. This case is close to the classical analysis in Meltzer and Richard (1981). We adopt their framework and use it to infer the properties of an optimal policy for the political

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class with heterogeneous population that can be modified with immigration policy.

2.1. **Notation and setup.**

2.1.1. *Population, skill, effort.* We consider a large population of agents. Each individual is endowed with a productive skill, in the set X, with generics element x. It can provide a productive effort e in a set $\mathbb{E} = \mathbb{R}$. Effort e costs to the individual an effort utility cost cost $C(e)$, where

Assumption 2.1. *C is a convex function, $C(0) = 0$, $C(e) = +\infty$ if $e < 0$.*

2.1.2. *Heterogeneous populations.* The distribution of skill in the population is described by a combination of normal random variables. More precisely, for any integer K we let the set $\mathbb{K} \equiv \{1, 2, \ldots, K\}$). For any vector $(\alpha, m, \sigma) \in \Delta(\mathbb{K}) \times \mathbb{R}_+^{2K}$, we write

$$X \sim \Phi(\cdot; \alpha, m, \sigma) \tag{1}$$

to indicate that X is the convex combinations with weights $((\alpha)_{k\in\mathbb{K}})$ of K normal random variables, each with mean m_k and standard deviation σ_k. When $K = 1$ this is a normal random variable, with $K = 2$ the population is the combination of two separate population, in proportions α_1 and α_2, each one with normal distribution $X_k \sim N(m_k, \sigma_k)$.

2.1.3. *Production.* The skill value x and the effort provided are not observable, the outcome (which is the gross income of the individual) is observed and can be taxed at a flat rate. A pair of skill and effort (x, e) produces a gross output of the individual equal to

$$y = f(x)e \tag{2}$$

where we assume

Assumption 2.2. *f is strictly increasing, $f(0) = 0$.*

2.1.4. *Allocation of tax revenues.* A tax rate is a real number $\tau \in [0, \overline{\tau}]$, with $\overline{\tau} \leq 1$. We assume that the revenues of the taxation is allocated equally over the entire population. The political class can appropriate a fraction $\gamma \in [0, 1]$ of the total revenue.

2.1.5. *Equilibrium Effort.* For a given tax rate τ and a given effort policy of all the types, namely a function $e^* : X \times [0, 1] \to \mathbb{E}$, the optimal effort of the skill type x is denoted $\hat{e}(x, \tau)$ and it solves:

$$\max_{e\in\mathbb{E}} \left((1 - \tau)f(x)e - C(e) + \tau(1 - \gamma)Y\right) \tag{3}$$

where

$$Y \equiv \int_{\mathbb{R}} e^*(\xi, \tau)f(\xi)d\Phi(\xi; \alpha, m, \sigma)$$

is the total product of the rest of the population at the effort policy e^* and distribution $\Phi(\cdot; \alpha, m, \sigma)$.

2.1.6. *Productivity at the optimal effort.* In the problem defined by (3) the term Y does not depend on the individual effort, so it does not enter in the maximization problem.

So the optimum in (3) reduces to finding the Fenchel conjugate of C:

$$C^*((1-\tau)f(x)) \equiv \max_{e\in\mathbb{E}} \left((1-\tau)f(x)e - C(e)\right). \tag{4}$$

which is the value under control of the individual, who has to decide the effort. That maximization problem also defines the effort policy function of each type, giving the function e^*.

To illustrate, if

$$C(e) = \frac{1}{2}e^2 \tag{5}$$

so $C^*((1-\tau)f(x)) = \frac{1}{2}\left((1-\tau)f(x)\right)^2$.

2.2. **The political class.** There is a political class that chooses α and γ to maximize the value of the product apppropriated.

2.3. **The game with majority voting.** The overall game follows these steps:

(1) The political class decides α through immigration policy; the parameters (m, σ) for the relevant sub-populations are given by history, culture and genetics.
(2) Taking that α as given, the political class then decides γ (the two choices may become real at different speed);
(3) Taking the values (α, γ) as given, the population decides an optimal effort anticipating the equilibrium (majority rule voting) tax rate, and then votes the tax rate;
(4) Production takes place, tax revenues are collected and the political class appropriates

3. Static Case with no effort choice

3.1. **Voting Equilibrium.** In this subsection we assume no effort is needed. In our general setup this is equivalent to assuming:

Assumption 3.1. *No effort choice:*

$$C(e) = 0 \text{ if } e \in [0,1], = +\infty \text{ otherwise.} \tag{6}$$

In this case the problem of the agent simplifies to choice of his favorite tax rate when his utility is given by:

$$(1-\tau)f(x) + \tau(1-\gamma)E(f(X)). \tag{7}$$

where X is *any* random variable (not necessarily normal). The following is clear:

Lemma 3.2. *Assume (3.1) (no effort choice). Then the equilibrium tax rate is either* 0 *or* $\overline{\tau}$*.*

Proof. This follows from the fact that the problem in (7) is linear in τ. $\square$

3.2. **No effort, no appropriation and homogeneous population.** Let us consider first the case of an homogeneous population, and no appropriation, which is the case that (including effort) Meltzer and Richard (1981) consider; so $K = 1$ and $\gamma = 0$. The nature of the voting equilibrium is simple when the production function f is either convex or concave:

Proposition 3.3. *Assume (3.1) (no effort choice), (2.2) (production function is strictly increasing), X a normal random variable, and $\gamma = 0$. Then, if f is strictly convex (concave) then $\tau^* = \overline{\tau}$ ($\tau^* = 0$ respectively).*

Proof. The agent of type x decides his favorite tax rate by solving (7) with $\gamma = 0$, so he checks the sign of $E(f(X)) - f(x)$. If this is larger than 0 then his favorite tax rate is $\hat{\tau}(x) = \overline{\tau}$, else it is 0. Since X is normal,

$$Median(X) = E(X). \tag{8}$$

If f is convex, and non linear, and X is normal (so the density is non-zero everywhere):

$$\begin{aligned} f(Median(X)) &= f(E(X)) \\ &< E(f(X)) \end{aligned}$$

and the opposite inequality if it is concave.

Since f is strictly increasing we have:

$$f(Median(X)) = Median(f(X)) \tag{9}$$

so the median voter chooses $\hat{\tau}(Median(X)) = \overline{\tau}$ if f is convex, and 0 if concave. $\square$

This proposition tells us that when productivity of intelligence is strictly increasing (that is f convex) then there is room for the political class to extract surplus from the public, in an homogeneous population. Now we show how they can do this optimally.

3.3. **Optimal appropriation.** The optimal appropriation rate is decided by the political class to maximize the product that will be appropriated. The objective function of the political class is to maximize the total slice of the revenues pie, at the equilibrium tax rate, that is the political class solves:

$$\max\{\gamma\tau^*(\gamma)E(f(X)) : \gamma \in [0, 1]\}. \tag{10}$$

Consider now X any random variable (not necessarily normal). Here is the optimal extraction rate:

Proposition 3.4. *Assume (3.1) (no effort choice), (2.2) (production function is strictly increasing), X a given random variable. Then*

(1) The optimal γ is

$$\hat{\gamma}(X) = \max\left\{1 - \frac{f(Med(X))}{E(f(X))}, 0\right\} \tag{11}$$

(2) The total payoff to the political class at the optimal choice is:

$$\overline{\tau}\max\{(E(f(X)) - f(Med(X))), 0\}. \tag{12}$$

Proof. The second claim follows immediately from the first.

In the case we are considering the only case in which the value to be maximized in (12) is strictly positive is when the equilibrium tax rate is $\overline{\tau}$ (from lemma (3.2). So the objective in (12) is equivalent to maximizing $\gamma\overline{\tau}E(f(X))$ under the constraint that $\overline{\tau} = \tau^*$. By lemma (3.2) this constraint is satisfied if the median voter prefers $\overline{\tau}$ to a zero tax rate. The median voter's income is the median income by (2.2). So he prefers $\overline{\tau}$, (from (7)) if and only if:

$$1 - \frac{f(Med(X))}{E(f(X))} \geq \gamma \tag{13}$$

hence the first claim follows as well. □

3.4. **Optimal immigration policy.** We have concluded in the previous section (proposition (3.4)) that the optimal extraction rate depends from the random variable X and the function f. Whether the political class can affect the function f is an interesting issue, but we abstract from it here.

Now we let the political class choose optimally the distribution of skills in the population, namely the random variable X. We consider first the case in which there are two sub-populations

We denote $\alpha \in [0, 1]$ the value of the fraction of low skill population. Since the maximum tax rate $\overline{\tau}$ is constant, the political class is trying to maximize: the difference:

$$\max\{E(f(X(\alpha))) - Med(f(X(\alpha))) : \alpha \in [0, 1]\} \tag{14}$$

where $X(\alpha)$ denotes the random variable obtained as a combination of of $X_i \sim N(m_i, \sigma), i = 1, 2$, with weight α on X_1.

The pattern of this variable as a function of the choice parameter α is first decreasing, then increasing. This pattern should not be surprising. To understand the reason behind the shape, one needs to begin by consider first that

$$E(f(X(\alpha))) = \alpha E(f(X_1)) + (1 - \alpha)E(f(X_2)) \tag{15}$$

The role of the median term $Med(f(X(\alpha)))$ is clear if one considers first the case in which the variables X_i are truncated so that their support does not overlap. One can then see that as we increase α the median first stay close to the value $f(m_2)$ (so the difference to be maximized is negative) and (at approximately $\alpha = 1/2$ in the symmetric case) the median switches top be close to $f(m_1)$ (so the difference to be maximized is positive).

3.5. **Estimated parameters.** We choose the parameters for the production function from an estimate of the effect of intelligence. Th best fit is an exponential

We consider a mixed population with two sub-populations, one with mean IQ = 100 and the other with mean IQ = 85, and consider all possible fractions between 0 (the entire population is normally distributed with mean 100) and 1.

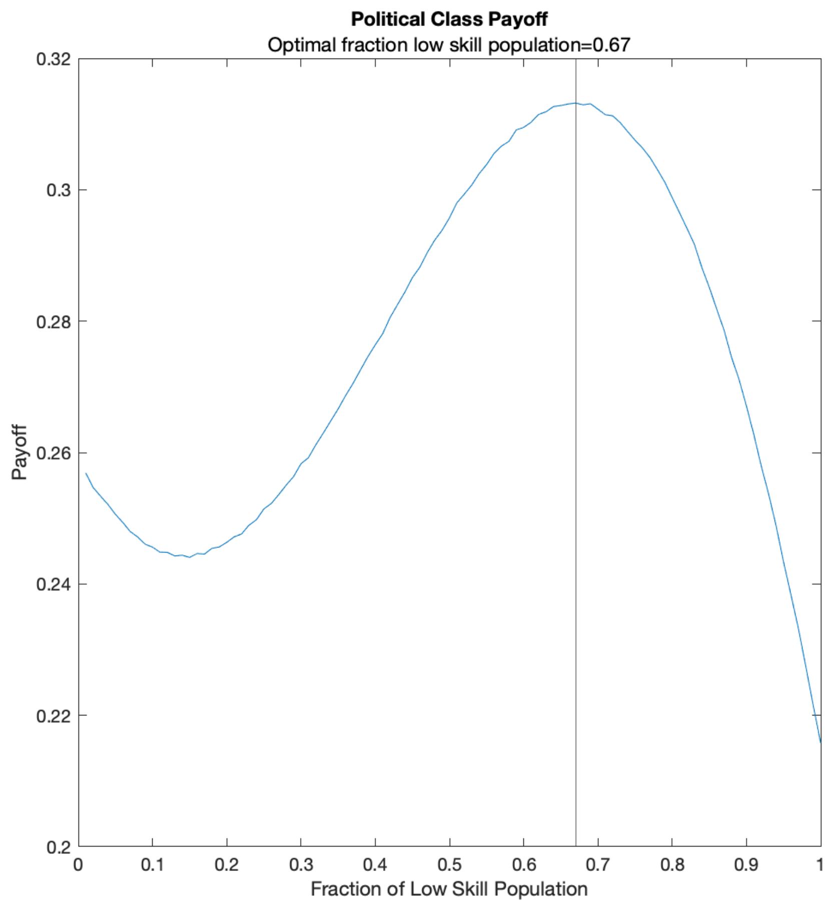

FIGURE 1. **Payoff of the political class.** On the x-axis the fraction of the low skill population. On the y-axis the payoff of the political class. The optimal fraction of low skill population is 0.68, indicated by the vertical bar.

4. Static Case with effort choice

We now consider the optimal population choice when agents choose an effort provision. To make the analysis more transparent we adopt the quadratic effort cost assumption in equation (5).

The optimal choice of the political class consists again of two steps, first for any given population composition choose the optimal extraction rate γ and then look for the optimal population choice. We begin with the choice of the optimal γ.

4.1. **Optimal extraction rate with effort.** The problem of the political class given α is choosing γ anticipating the two sequential choices of the agents, who will first choose the optimal effort for a give tax rate and extraction rate, and then at the optimal effort, compute the favorite tax and vote.

We denote the optimal effort at (x, τ, γ) as $\hat{e}(x, \tau)$; the policy is independent of γ. We also write $\hat{\tau}(x, \tau, \gamma)$ as the favorite taxation.

The political class, given a distribution with density $\phi(\cdot; \alpha)$ on population skill solves:

$$\max \left\{ \gamma\tau^* E_{\phi(\cdot;\alpha)} \left(\hat{e}(X, \tau^*, \gamma) f(X) \right) : \tau^* = \hat{\tau}(Med(X), \tau^*, \gamma) \right\} \tag{16}$$

4.2. **Optimal effort and tax choice.** We have seen already that the optimal effort with quadratic cost is

$$\hat{e}(x, \tau) = (1 - \tau) f(x)^2. \tag{17}$$

Substituting (17) into the favorite taxation problem of the agent we get the problem:

$$\max \left\{ A(x)(1 - \tau)^2 + B(\gamma, \alpha)\tau(1 - \tau) : \tau \in [0, 1] \right\}, \tag{18}$$

where

$$A(x) \equiv \frac{f(x)^2}{2}, \tag{19}$$

and

$$B(\gamma, \alpha) = (1 - \gamma) E_{\phi(\cdot,\alpha)} f(X)^2. \tag{20}$$

Proposition 4.1. *The favorite tax rate is:*

$$\hat{\tau}(x, \tau, \gamma) = \frac{(B(\gamma, \alpha) - 2A(x))^+}{2(B(\gamma, \alpha) - A(x))}$$

Proof. The proof is algebra. □

Note (4.1) implies that the favorite tax rate is positive when $B(\gamma, \alpha) - 2A(x) > 0$, that is when $f(x) < \sqrt{(1 - \gamma) E_{\phi(\cdot,\alpha)} f(X)^2}$, and since $\sqrt{E_{\phi(\cdot,\alpha)} f(X)^2} \geq E_{\phi(\cdot,\alpha)} f(X)$ the wedge for the political class is larger. On the other hand, the term $1 - \gamma$ is now taken with the square root.

Overall the pattern of choices is similar to the no effort case.

References

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